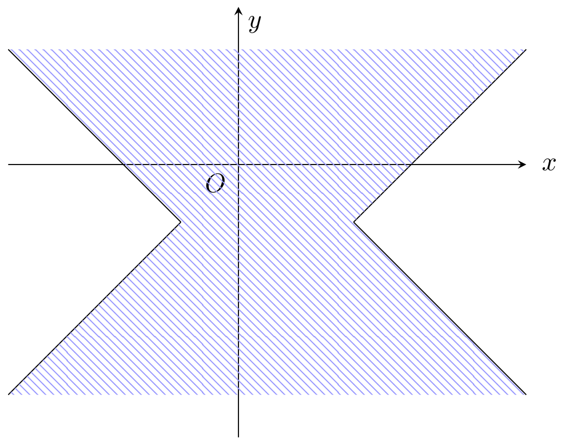
P3 Assignment Solution

Assignment 1.

1. (a) $(-\infty, -\frac{1}{5}] \cup [1, \infty)$
   (b) $(-\frac{3}{4}, -\frac{1}{2})$
2. 3 or $\frac{-1-\sqrt{41}}{2}$.
3. (a) $a = \frac{5}{3}$, $b = -\frac{2}{3}$
   (b) $\frac{8}{3}x + \frac{16}{3}$
4. (a) $k = -15$
   (b) $(-\infty, 1) \cup (2, \infty)$
5. The region is as follows:



Assignment 2.

1. $1 - \frac{3}{8}x - \frac{37}{128}x^2 + \frac{57}{1024}x^3 \cdots$
2. (a) omit
   (b) $\frac{1}{2} + \frac{1}{16}x^2 + \frac{7}{256}x^4 + \cdots$
3. (a) $a = 2$
   (b) $-\frac{105}{64}$
4. (a) $f(x) = \dfrac{\frac{1}{4}}{x+1} + \dfrac{\frac{3}{4}}{x-1} + \dfrac{\frac{1}{2}}{(x-1)^2}$
   (b) $x^2 + x^3 + 2x^4$
5. $\frac{27}{16}$, no terms in the expansion of $\left(1 + \frac{1}{3}x\right)^{\frac{1}{2}}$ has the term $x^{-\frac{5}{2}}$

Assignment 3.

1. $x > \log_{0.8} 0.5 = 3.11$
2. $\dfrac{x}{y} = \dfrac{\ln 2.5}{\ln 1.25} = 4.11$
3. $z = \dfrac{y+2}{y^2}$
4. Min= $\dfrac{3}{4}$, max= 57.
5. (a) $x = \dfrac{\ln 2}{\ln 4 - \ln 3} = 2.409.$
   (b) $x = 0.802$
   (c) $x = \pm 1.585$
6. Domain: $x \in \mathbb{R}$, range: $-1 < y < 1$.

Assignment 4.

1. $x = -49.1°$ or $130.9°$.
2. (a) omit
   (b) 0.955 or 5.33
3. $x = 48.2°$ or $311.8°$ or $120°$ or $240°$
4. (a) $R = 13$, $\alpha = 22.6.2°$
   (b) $17.1°$ or $297.7°$
5. (a) omit
   (b) $x = \frac{\pi}{8}$ or $\frac{5}{8}\pi$

Bonus question:

1. (a) 0.894, 0.0599 or -0.835
   (b) $\pm\frac{11}{2}$.
2. $4 : 5 : 6$

Assignment 5.

1. e^{-1}
2. $y - e^2 = 3e(x - e)$
3. $y = -\frac{1}{\pi}(x - \frac{\pi}{2})$
4. $x = \frac{\pi}{4}$
5. (a) $2^{-\frac{3}{2}}$

   (b) $-2^{-\frac{3}{2}}$
   (c) $-\frac{4\sqrt{5}}{25}$

6. $x^x(\ln x + 1)$

Assignment 6.

1. $y - 2 = x - 1$
2. $y - 1 = -2(x - 1)$
3. $(1, 0)$ and $(4, 12e^{-4})$
4. Max: $(\frac{\pi}{4}, 3)$, $(\frac{3}{4}\pi, -1)$; Min $(\frac{11}{12}\pi, -\frac{3}{2})$, $(\frac{7}{12}\pi, -\frac{3}{2})$
5. (a) Omit
   (b) $x^4 + y^2 = 1$
   (c) $(\frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2})$, $(\frac{\sqrt{2}}{2}, -\frac{\sqrt{3}}{2})$

Assignment 7.

1. $[x\ln 2x - x]_1^a = a\ln 2a - a - \ln 2 + 1$
2. (a) $u = \tan x$, then $\frac{\mathrm{d}u}{\mathrm{d}x} = \sec^2 x$, then it equals to $\displaystyle\int_0^1 u^n\,\mathrm{d}u = \frac{1}{n}$
   (b) i. $= \displaystyle\int_0^{\frac{1}{4}\pi} \sec^2 x(\sec^2 x - 1)\,\mathrm{d}x = \int_0^{\frac{1}{4}\pi} (1 + \tan^2 x)\tan^2 x\,\mathrm{d}x = \int_0^{\frac{1}{4}\pi} \tan^2 x + \tan^4 x\,\mathrm{d}x = \frac{1}{3}$
      ii. Split into $t^9 + t^7 + 4(t^7 + t^5) + t^5 + t^3$, final answer $\frac{25}{24}$
3. (a) 0.685
   (b) $\frac{8}{15}$
4. $\ln(\frac{16}{9})$

Assignment 8.

1. (a) Omit
   (b) $\dfrac{10u}{(3-u)(2+u)} = \dfrac{6}{3-u} + \dfrac{-4}{2+u}$
2. (a) $f(x) = \dfrac{3}{3x+2} + \dfrac{-x+3}{x^2+4}$
   (b) $\dfrac{3}{2}\ln 2 + \dfrac{3}{8}\pi.$
3. (a) $y = x - 1$
   (b) $\dfrac{1}{4}(\mathrm{e}^2 - 1)\pi$
4. $\dfrac{x}{\sqrt{x^2+1}}\ln x - \ln\left|x + \sqrt{1+x^2}\right| + C.$

Assignment 9.

1. (a) Omit

   (b) Let $f(x) = \dfrac{x}{3} + 2 - e^{-x}$, then $f(-1) = -1.05 < 0$, and $f(0) = 1 > 0$, so there is $x \in (-1, 0)$ such that $f(x) = 0$, it follows that $\dfrac{x}{3} + 2 = e^{-x}$ has a root lies between -1 and 0.

   (c) Suppose $x_n \to \alpha$, then $x_{n+1} \to \alpha$, hence $\alpha = \ln 3 - \ln(\alpha + 6)$, then $e^{\alpha} = \frac{3}{\alpha+6}$, it follows $e^{-\alpha} = \frac{\alpha+6}{3}$, which implies that $\frac{\alpha}{3} + 2 = e^{\alpha}$.

   (d) -0.59.

2. (a) Omit

   (b) Omit

   (c) 5.64

3. (a) $1 + x^3 - \frac{1}{2}x^6$

   (b) 1.00

   (c) Omit

   (d) $-0.5 < x < 0$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} < 0$, for $0 < x < 0.5$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} > 0$

4. (a) Omit

   (b) $\frac{26}{3}$

   (c) 8.61

   (d) greater.

Assignment 10.

1. $\tan y = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$.

2. $y = e^{x \ln x - x}$. $y = \frac{1}{e}$.

3. $y = \dfrac{(1+x^2)^3}{4} - 1$.

4. (a) $P = \dfrac{KP_0e^{rt}}{K - P_0 + P_0e^{rt}}$.

   (b) $P \to K$.

5. (a) Omit

   (b) $V = 18 - 18e^{-\frac{t}{2}}$.

   (c) $t = 2\ln 2$.

Assignment 11.

1. (a) $\begin{pmatrix} 2 \\ 6 \\ 3 \end{pmatrix}$

   (b) $\frac{1}{7}\begin{pmatrix} 2 \\ 6 \\ 3 \end{pmatrix}$

2. First set up the equations involving t and s, then solve it. $s = 1$ and $t = -3$. The position vector of the point of intersection is $\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$

3. (a) Solve the equation, $s = \frac{2}{3}$ and $t = -\frac{7}{3}$. But $4 - \frac{2}{3} \neq 1 - \frac{7}{3}$.

   (b) The position vector of P is $\begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$

   (c) $t = -2$, the proof is omit.

4. (a) $45.6°$
   (b) Omit
   (c) Omit

Assignment 12.

1. $\frac{25}{13} - \frac{5}{13}\mathrm{i}$
2. $1 - 2\mathrm{i}$.
3. $2\sqrt{2} + 3$
4. $\frac{\pi}{2} - \theta + k \cdot 2\pi$
5. (a) $11 - 60\mathrm{i}$.
   (b) $6 - 5\mathrm{i}$ or $-6 + 5\mathrm{i}$.
   (c) $3 - 2\mathrm{i}$ or $\frac{1}{2}\mathrm{i}$.

Assignment 13.

1. 2 or $-3 - \mathrm{i}$.
2. $|Z| = 2$ and $arg(Z) = \frac{2}{3}\pi$, $-\frac{1}{4} - \frac{\sqrt{3}}{4}\mathrm{i}$.
3. Omit, $\sqrt{2} + (2 + \sqrt{2})\mathrm{i}$.
4. (a) $|Z - \mathrm{i}| = |\sin\theta + \mathrm{i}(-\cos\theta)|$
   (b) Omit
   (c) $|Z| = \sqrt{2 - 2\cos\theta}$
   (d) $\frac{\theta}{2}$.